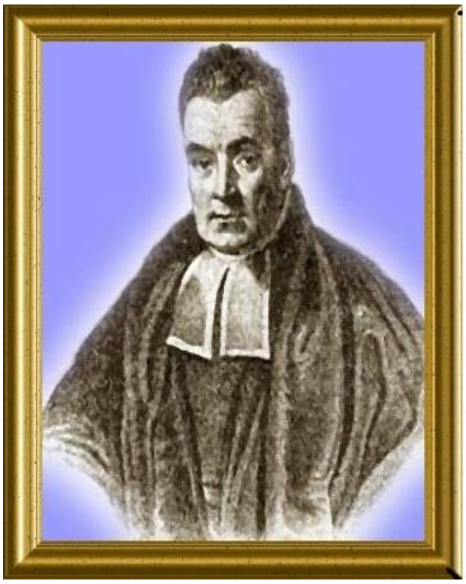
Probability

Daniel Bernoulli : (8 February 1700 – 17 March 1782) was a Swiss mathematician and physicist and was one of the many prominent mathematicians in the Bernoulli family. He is particularly remembered for his applications of mathematics to mechanics, especially fluid mechanics, and for his pioneering work in **probability and statistics.**



Thomas Bayes : (1701 - 1761) was an English statistician, philosopher and Presbyterian minister who is known for having formulated a specific case of the theorem that bears his name: **Bayes' theorem**. Bayes never published what would eventually become his most famous accomplishment; his notes were edited and published after his death by Richard Price.

Probability

Introduction

We studied Permutation and combination in Std. XI. Let us briefly review these concepts because they have been used in this chapter.

1. Permutation : If n objects are given and we have to arrange r ($r \le n$) out of them such that the order in which we are arranging the objects is important, then such an arrangement is called permutation of n objects taking r at a time. This is denoted by ${}^nP_r$.

2. Combination : If n objects are given and we have to choose r ($r \le n$) out of them such that the order in which we are choosing the objects is not important, then such a choice is called combination of n objects taking r at a time. This is denoted by ${}^nC_r$.

3. Factorial Notation : If $n \in N$, then the product $1 \times 2 \times 3 \times \ldots \times n$ is defined as factorial n which is denoted by n! or $\underline{|n}$ i.e., $n! = 1 \times 2 \times 3 \times \ldots \times n$.

We also define $0! = 1$ Note that : $n! = n(n-1)!$

4. Formula for ${}^nP_r$ and ${}^nC_r$:

$$
\begin{aligned}
{}^nP_r &= \frac{n!}{(n-r)!} \ (r \le n) \\
&= n(n-1)(n-2)\ldots(n-r+1)
\end{aligned}
$$

$$\text{e.g.} = {}^{10}P_3 = \frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7!}{7!} = 720 \ \text{OR}$$

$${}^{10}P_3 = 10 \times 9 \times 8 = 720$$

$$
\begin{aligned}
{}^nC_r &= \frac{n!}{r!(n-r)!} (r \le n) \\
&= \frac{n(n-1)(n-2)\ldots(n-r+1)}{1.2.3.\ \ldots.\ r}
\end{aligned}
$$

$$
\begin{aligned}
\text{e.g. } {}^{10}C_3 &= \frac{10!}{3!7!} \\
&= \frac{10 \times 9 \times 8 \times 7!}{3 \times 2 \times 1 \times 7!} = 120 \qquad \text{or}
\end{aligned}
$$

$${}^{10}C_3 = \frac{10 \times 9 \times 8}{1 \times 2 \times 3} = 120$$

$$\text{Note: } {}^nC_r = {}^nC_{n-r}$$

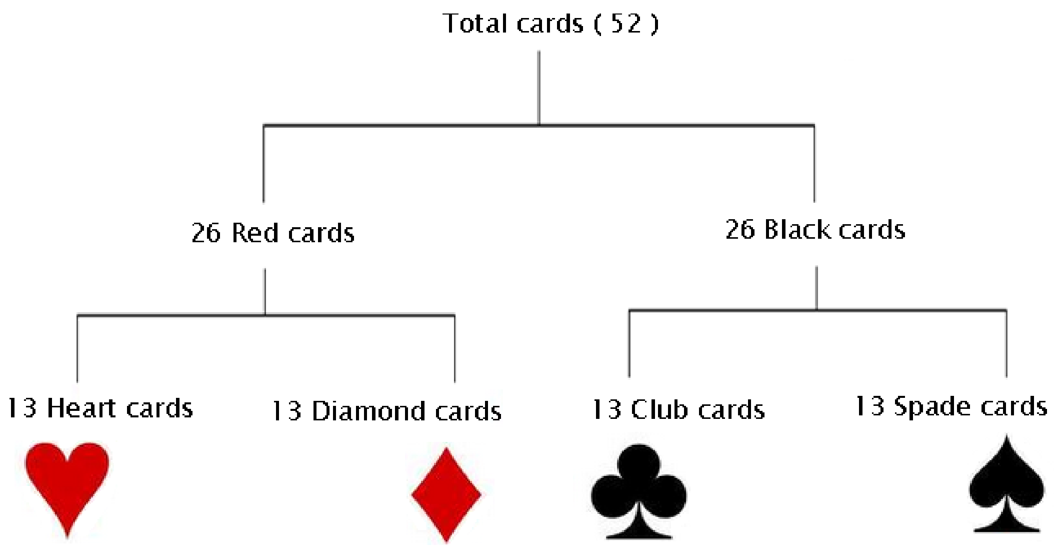
$$
\begin{aligned}
\text{e.g. } {}^{20}C_{18} &= {}^{20}C_{20-18} = {}^{20}C_2 \\
&= \frac{20 \times 19}{1 \times 2} = 190.
\end{aligned}
$$

5. Fundamental Principle :

If one thing can be done in m different ways and after it has been done by any one of these m ways, if the second thing can be done in n different ways, then the total number of ways of doing both the things together is m.n and the total number of ways of doing any one of the two is m + n.

This principle can be extended for any number of things.

Playing Cards



i. There are 4 suits, each contains Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king, i.e., 13 cards.

ii. There are 13 denominations, each contains heart, diamond, spade and club, i.e., 4 cards.

iii. Total number of face cards or picture cards is 12.

Concept of Probability

When a quadrilateral is given, without knowing the four angles, we can say definitely that the sum of measures of the angles must be 360^0. Similarly when we add two odd integers, we are sure that the sum is even number though we do not know the exact sum. But if we tossed a coin, it is not sure whether head or tail will turn up. It may turn up a head or tail. Similarly when a six faced die is thrown, the possible number on upper face is either 1, 2, 3, 4, 5, 6. It is not possible to state which of these will turn up.

Thus we are referring to the situation which may or may not occur. In fact, the above examples involve the degree of uncertainty.

It is possible to express numerically this uncertainty due to chance. Thus, in the case of normal coin head and tail have equal chance of occurrence. In the case of throwing a die, all the six numbers have equal chances of turning up. These are the some examples of probability. To understand the methods for finding the probability, in general, we need to learn certain basic concepts.

Definitions and Types of Events

Definitions

1. Random Experiment : If an act or an experiment has more than one possible results which are known in advance and it is not possible to predict which one is going to occur, then such an experiment is called a random experiment.

   The following are some random experiments :

   (1) Tossing of a coin

   (2) Throwing a six-faced die

   (3) Drawing two cards from a well-shuffled pack of cards.

   (4) Three persons are selected out of 18 persons to form a committee.

   (5) A ball is drawn from a bag containing 13 balls.

2. Outcome : The result of a random experiment is called an outcome.

3. Sample space : The set of all possible outcomes of a random experiment is called a sample space and its elements are called sample points.

   A sample space is usually denoted by S.

   e.g. (i) When a fair coin is tossed, then either Head or Tail will turn up. Hence S = {H, T}. S contains 2 sample points.

   (ii)When a six-faced die is thrown, then only one of 1, 2, 3, 4, 5, 6 will turn up. Hence S = {1, 2, 3, 4, 5, 6}. S contains 6 sample points.

4. Event : Any subset of a sample space is called an event.

If an event contains only one sample point, then it is called a simple event or an elementary event.

If an event is the empty set (i.e., it does not contain any sample point) then it is called an impossible event.

The sample space S is a subset of itself. Hence it is also an event. This event is called a certain event or a sure event, since it is sure to occur.

5. Complementary Event : Let A be an event in a sample space S. Then A is a subset of S. We can hence think of the complement of A in S, i.e. S – A. This is also a subset of S and hence an event in S. This event is called the complementary event of A and is denoted by $\overline{A}$ or A'.

Now, suppose S contains n sample points, A contains m sample points. Then A' will contain n – m sample points.

Probability of an Event

Definition

Let A be an event in a sample space S. Then the probability of the event A denoted by P(A) is defined as,

$$P(A) = \frac{\text{Number of sample points in A}}{\text{number of sample points in S}} = \frac{n(A)}{n(S)}.$$

Theorem

If E is an event of a sample space S, prove that 0 ≤ P(E) ≤ 1 and P(E') =1 – P(E), where E' is the complementary event of E.

Proof : Suppose the sample space S contains n sample points and the event E contains m sample points. Then $P(E) = \frac{m}{n}$.

$$\text{Now } 0 \le m \le n \quad \therefore 0 \le \frac{m}{n} \le 1$$

$$\therefore 0 \le P(E) \le 1.$$

Further E' contains n – m sample points.

$$\therefore P(E') = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(E)$$ Thus P(E') = 1 – P(E).

Two or more events

Let A and B be two events in sample space S. Then A and B are subsets of S. Hence $A \cup B$ and $A \cap B$ are also subsets of S. Therefore $A \cup B$ and $A \cap B$ are also events in S.

$A \cup B$ (or $A+B$) represents the occurrence of either A or B or both A and B.

$A \cap B$ represents the occurrence of both A and B.

Mutually exclusive Events : If A and B are disjoint then $A \cap B$ does not contain any sample point.

$\therefore$ $A \cap B = \emptyset$ and $n(A \cap B) = 0$

$\therefore$ $P(A \cap B) = 0$

Such events are called mutually exclusive events.

In this case if the event A occurs, then the event B cannot occur and if B occurs, then A cannot occur.

Exhaustive Events : Suppose A and B are two events in sample space S and $A \cup B = S$.

Then A and B are said to be exhaustive events.

For any event A,

$A \cap A' = \emptyset$ and $A \cup A' = S$

Hence A and A' are mutually exclusive and exhaustive.

We are interested in finding the probability of the event $A \cup B$ which we do in the following theorem called the Addition Theorem.

Addition Theorem

Theorem

If A and B are two events of sample space S, prove that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Hence deduce that $P(A') = 1 - P(A)$, where A and A' are complementary events of each other. (March '96– '06; Oct. '96)

Proof : Suppose

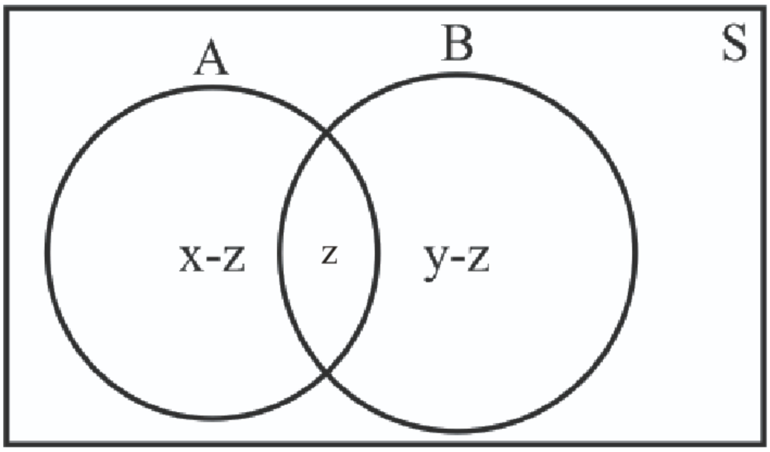
n = number of sample points in the sample space S

x = number of sample points in the event A

y = number of sample points in the event B

z = number of sample points in $A \cap B$.

Then $P(A) = \frac{x}{n}$, $P(B) = \frac{y}{n}$, $P(A \cap B) = \frac{z}{n}$.



Now from the Venn diagram, it is clear that number of sample points in $A \cup B$

$= (x - z) + z + (y - z) = x + y - z$

$$\therefore P(A \cup B) = \frac{x+y-z}{n} = \frac{x}{n} + \frac{y}{n} - \frac{z}{n}$$

$$= P(A) + P(B) - P(A \cap B)$$

Second Part : A' is the complementary event of A

$\therefore A \cup A' = S$ and $A \cap A' = \varnothing$

$\therefore P(A \cup A') = P(S) = 1$ and $P(A \cap A') = P(\varnothing) = 0$

Using these in the addition theorem, we get,

$P(A \cup A') = P(A) + P(A') - P(A \cap A')$

$\therefore 1 = P(A) + P(A')$ $\therefore P(A') = 1 - P(A)$.

Remarks :

1. If the events A and B are mutually exclusive, then $A \cap B = \varnothing$ $\therefore P(A \cap B) = P(\varnothing) = 0$.

   Hence in the case of mutually exclusive events, the addition theorem becomes $P(A \cup B) = P(A) + P(B)$

2. De Morgan's Law :

If A and B are subsets of the universal set S, then

(i) $(A\cup B)' = A'\cap B'$ and (ii) $(A\cap B)' = A'\cup B'$.

$\therefore$ $P(A'\cap B') = P(A\cup B)' = 1 - P(A\cup B)$ and

$P(A'\cup B') = P(A\cap B)' = 1 - P(A\cap B)$.

Theorem

If A, B, C are three mutually exclusive events of a sample space S, prove that

$P(A\cup B\cup C) = P(A) + P(B) + P(C)$.

Proof :

By the addition theorem,

$P(A\cup B\cup C) = P[A\cup(B\cup C)]$

$= P(A) + P(B\cup C) - P[A\cap(B\cup C)]$

$= P(A) + P(B) + P(C) - P(B\cap C) - P[(A\cap B) \cup (A\cap C)]$

$= P(A) + P(B) + P(C) - P(B\cap C) - P(A\cap B) - P(A\cap C) + P[(A\cap B) \cap (A\cap C)]$

$= P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$ ... (1)

Now A, B, C are mutually exclusive events.

$\therefore$ $A\cap B = \varnothing$, $B\cap C = \varnothing$, $A\cap C = \varnothing$, $A\cap B\cap C = \varnothing$

$\therefore$ $P(A\cap B) = P(\varnothing) = 0$, $P(B\cap C) = 0$, $P(A\cap C) = 0$

and $P(A\cap B\cap C) = 0$

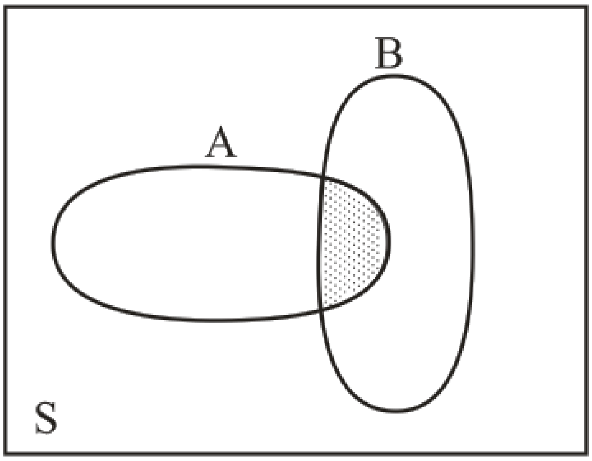
$\therefore$ from (1), if A, B, C are mutually exclusive events, then

$P(A\cup B\cup C) = P(A) + P(B) + P(C)$.

Conditional Probability and Independent Events

Sometimes the probability of a given event depends on the occurrence or non-occurrence of some other event.

Suppose A and B are two events in a sample space S.



Let n = number of sample points in S

m_1 = number of sample points in A

m_2 = number of sample points in B

m_{12} = number of sample points in $A \cap B$.

Then,

$$P(A)=\frac{m_1}{n},\ \ P(B)=\frac{m_2}{n} \text{ and } P(A \cap B)=\frac{m_{12}}{n}$$

Now, suppose that the event A has taken place. On this assumption, the m_1 sample points of A constitute the sample space for other events. In particular, the event B in this sample space occurs along with A. Hence in this sample space occurs along with A , m_{12} sample points belong to B also. These are the sample points in $A \cap B$.

The probability of $A \cap B$, (i.e., of B) in the sample space A is $\frac{m_{12}}{m_1}$. This is the probability of B under the assumption that A takes place. It is denoted by P (B/A) and is called the conditional probability of B given that A takes place.

$$\therefore P(B/A)=\frac{m_{12}}{m_1}=\frac{n(A \cap B)}{n(A)}, \text{ provided } n(A) \neq 0$$

Similarly, P(A/B) is the conditional probability of A given that B has taken place and

$$\therefore P(A/B) = \frac{m_{12}}{m_2} = \frac{n(A \cap B)}{n(B)}, \text{ provided } n(B) \neq 0$$

Two events A and B are said to be independent, if $P(A/B) = P(A)$ and $P(B/A) = P(B)$.

Multiplication Theorem

Theorem

If A and B are two events, prove that $P(A\cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$.

Proof : By definition,

$$P(B/A) = \frac{n(A \cap B)}{n(A)}, \text{ provided } n(A) \neq 0$$

$$= \left[\frac{n(A \cap B)}{n(S)}\right] \bigg/ \left[\frac{n(A)}{n(S)}\right]$$

where S denotes the sample space and $n(S) \neq 0$

$$\therefore P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$\therefore P(A\cap B) = P(A) \cdot P(B/A)$

Similarly, we can prove that

$P(A\cap B) = P(B) \cdot P(A/B)$

Hence, $P(A\cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$

Remarks

1. If A and B are independent, then

   $P(B/A) = P(B)$ and $P(A/B) = P(A)$ $\quad \therefore P(A\cap B) = P(A) \cdot P(B)$.

2. In general, if $A_1, A_2, A_3, \ldots, A_n$ are n independent events, then probability of simultaneous occurrence of these n events is

   $P(A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n) = P(A_1) \cdot P(A_2) \cdot P(A_3) \cdot \ldots \cdot P(A_n)$.

3. If A and B are mutually exclusive events, then

   $P(A\cap B) = 0 \quad \therefore P(A/B) = P(B/A) = 0$.

Odds in Favour And Against

If the probability of occurrence of an event A is p then the probability of non-occurrence of A is 1− p.

∴ the odds in favour of A is given by p : (1− p).

The odds against A is given by (1− p) : p.

For example : If P(A) = $\frac{1}{3}$, then P(A') = 1 - P(A)

$$=1-\frac{1}{3}=\frac{2}{3}$$

∴ the odds in favour of A is given by $\frac{1}{3}:\frac{2}{3}$, i.e. $1 : 2$

∴ The odds against A is given by $\frac{2}{3}:\frac{1}{3'}$ i.e., $2 : 1$.

Conversely :

i. If odd in favour of an event A are a : b, then the probability of the occurrence of A is $\frac{a}{a+b}$ and the probability of non-occurrence of A is c

ii. If odds against an event A are a : b, then the probability of the occurrence of A is $\frac{b}{a+b}$ and the probability of non-occurrence of A is $\frac{a}{a+b}$

Bayes' Theorem

Theorem

If $A_1, A_2, A_3, \ldots, A_n$, are n mutually exclusive and exhaustive events from the sample space S, B is any other event from S and if probability of occurrence of A_i's and probability of occurrence of B given that A_i, i = 1, 2, 3, …, n has occurred are known, then probabilities of occurrence of A_i's given that B has occurred are given by

$$P(A_i / B)=\frac{P(A_i).P(B / A_i)}{\sum_{i=1}^{n} P(A_i).P(B / A_i)};\ i=1, 2, 3, \ldots, n.$$

Note : $P(A_i)$; i = 1, 2, 3, …, n are known as a priori or simply prior probabilities and $P(A_i/B)$ are called posterior probabilities.

Solved sums

1. A card is drawn at random from an ordinary pack of 52 playing cards. State the number of elements in the sample space if consideration of suits.
   a. is not taken into account.
   b. is taken into account.

Sol: a. is not taken into account.

If consideration of suits is not taken into consideration, then a card is selected at random from 52 cards.

i.e. S = To select one card out of 52.

$\therefore$ n (S) = ${}^{52}C_1$ = 52

(b) Is taken into account.

Sol: If consideration of suits is taken into account then the desired suits is separated and a card is selected from that suit.

i.e. S = A card is selected at random from the

Desired suit containing 13 cards, (Either spades or clubs or hearts or diamonds).

$\therefore$ n (S) = ${}^{13}C_1$ = 13

2. Write all the possible outcomes for the occurrence of a number 4 at least once when a fair die is tossed two times.

Sol: Let S = A die is rolled twice so that 4 occurs at least once.

i.e. Either 4 occurs in 1st toss or 2nd toss or in both.

= {(4,1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (1, 4), (2, 4), (3, 4), (5, 4), (6,4)}

$\therefore$ n (S) = 11.

3. Four cards are drawn from a pack of 52 cards. What is the probability that
   a. 3 are Kings and 1 is Jack?
   b. all the cards from different suit?
   c. at least one heart?
   d. all 4 are clubs and one of them is jack?

Sol: Let S = 4 cards are drawn at random from a pack of 52 cards.

$\therefore$ n (S) = ${}^{52}C_4$

(a) Let event A

= 3 cards are kings and 1 is jack.

= To draw 3 king from 4 and 4 and 1 jack from 4.

$\therefore$ n (A) = $^4C_3 \times {}^4C_1$

$$\therefore \ P(A) = \frac{n(A)}{n(S)} = \frac{^4C_3 \times {}^4C_1}{^{52}C_4}$$

(b) Let event B

= All 4 cards are different suits.

= To draw 1 card each from every suit of 13 cards

$\therefore$ n (B) = $^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$$\therefore \ P(B) = \frac{n(B)}{n(S)} = \frac{^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{^{52}C_4}$$

(c) Let event C'

= At least one heart.

= To draw 4 cards with at least one heart.

$\therefore$ Complement, C'

= To draw all 4 cards from non-hearts (39 cards)

$\therefore$ n (C') = $^{39}C_4$

$$\therefore \ P(C') = \frac{n(C')}{n(S)} = \frac{^{39}C_4}{^{52}C_4}$$

$\therefore$ Required Probability,

$$\therefore \ P(C) = 1 - P\ (C') = 1 - \frac{^{39}C_4}{^{52}C_4}$$

(d) Let event D

= All 4 cards are clubs including their jack.

= To draw the jack club and 3 other clubs from 12

$\therefore$ n (D) = 1 $\times$ $^{12}C_3$

$$\therefore \ P(D) = \frac{n(D)}{n(S)} = \frac{^{12}C_3}{^{52}C_4}$$

4. From a group of 8 boys and 5 girls, a committee of 5 is to be formed. Find the probability that the committee contains.

a. 3 boys and 2 girls

b. at least 3 boys

Sol: Let S = To select 5 from a group of 8 boys, 5 girls (total 13).

$\therefore$ n (S) = $^{13}C_5$

(a) Let event A

= To select 3 boys from 8 and 2 girls from 5.

$\therefore$ n (A) = $^8C_3 = \times {}^5C_2$

$\therefore$ $P(A)=\frac{n(A)}{n(S)}=\frac{{}^8C_3\times{}^5C_2}{{}^{13}C_5}$

(b) Let event B

= To select 5 containing at least 3 boys.

= To select 3 boys 2 girls or 4 boys 1 girl or all 5 boys.

$\therefore$ n (B) = $^8C_3 \times {}^5C_2 + \times {}^8C_4 \times {}^5C_1 + {}^8C_5$

$\therefore$ $P(B)=\frac{n(B)}{n(S)}$

$=\frac{{}^8C_3\times{}^5C_2\times{}^8C_4\times{}^5C_1+{}^8C_5}{{}^{13}C_5}$

5 The letters of the word LOGARITHM are arranged at random. Find the probability that

a. Vowels are always together

b. Vowels are never together.

c. Exactly 4 letters between G and H

d. Begin with O and end with T.

e. Start with vowel and end with consonant.

Sol: The word LOGARITHM contains 9 distinct letters consisting of 3 vowels and 6 consonants.

Since the letters are to be rearranged in linear places, each arrangement is a permutation.

Let S = To rearrange all 9 letters in 9 linear places.

$\therefore$ n (S) = $^9P_9 = 9!$

(a) Let event A

= In the arrangement, all 3 vowels are together.

= To arrange 1 block of vowels + 6 consonants in 7 places and 3 vowels are rearranged in the block of 3 places.

$\therefore$ n (A) = $7! \times 3!$

$\therefore$ $P(A)=\frac{n(A)}{n(S)}=\frac{7!\,3!}{9!}=\frac{7!3\times2\times1}{9\times8\times7!}=\frac{1}{12}$

(b) Let event B

= In the arrangement, no two vowels are together.

= 6 consonants to be arranged in 6 linear places and 13 consonants are inserted in any 3 of 7 alternate place.

(2 end + 5 interior)

$\therefore$ n (B) = $6! \times {}^7C_3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6! \times {}^7C_3}{9!}$$

$$= \frac{6! \times 7 \times 6 \times 5}{9 \times 8 \times 7 \times 6!} = \frac{5}{12}$$

(c) Let event C

= In the arrangement, there are exactly 4 letters between G and H.

= Such an arrangement is possible only if G and H occupy

1^{st} and 6^{th}

2^{nd} and 7^{th}

3^{rd} and 8^{th}

4^{th} and 9^{th} places and remaining 7 letters can be arranged in remaining 7 places.

$\therefore$ n (C) = 4 × 7!

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{4 \times 7!}{9!}$$

$$= \frac{4 \times 7!}{9 \times 8 \times 7!} = \frac{1}{18}$$

(d) Let event D

= The arrangement begins with O and ends with T.

= 1^{st} place is occupied by O and 9^{th} place is occupied by T, remaining 7 letters are arranged in 7 remaining places.

$\therefore$ n (D) = 1 × 7! × 1 = 7!

$$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{7!}{9!}$$

$$= \frac{7!}{9 \times 8 \times 7!} = \frac{1}{72}$$

(e) Let event E

= The arrangement starts with a vowels and ends with a consonant.

= 1^{st} place is occupied by any one of 3 vowels and 9^{th} place is occupied by any one of 6 consonants, remaining 7 places are occupied by remaining 7 letters.

$\therefore$ n (E) = 3 × 7! × 6

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3 \times 7! \times 6}{9!}$$

$$= \frac{3 \times 7! \times 6}{9 \times 8 \times 7!} = \frac{1}{4}$$

6. 5 letters are to be posted in 5 post boxes, if any number of letters can be posted in all 5 post boxes, what is the probability that each box contains only one letter?

Sol: Since any number of letters can be posted with of the post boxes, repetition is allowed.

i.e. each box is available for all 5 letters.

$\therefore$ Let S = To post 5 letters in 5 boxes with repetition.

$\therefore$ n (S) = $5 \times 5 \times 5 \times 5 \times 5 = 5^5$

Let event A

= Each box contains only one letters.

= To post one letter in each of 5 boxes in a linear order.

$\therefore$ n (A) = ${}^5P_5 = 5!$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{5!}{5^5}$$

$$\therefore \quad = \frac{5\times4\times3\times2\times1}{5\times5^4} = \frac{24}{625}$$

7. What is the probability of getting 9 cards of the same suit in one hand at a game of bridge?

Sol: In a game of bridge, 13 cards are distributed from a pack of 52 cards.

Let S = To select any 13 cards from 52 cards.

$\therefore$ n (S) = ${}^{52}C_{13}$

Let event A

= A particular player gets a hand containing 9 cards of the same suit and 4 other cards.

= A suit out of 4 suits, 9 cards from that suit (13 cards) and 4 from remaining 39 cards.

$\therefore$ n (A) = ${}^4C_1 \times {}^{13}C_9 \times {}^{39}C_4$

$$\therefore \ P(A) = \frac{n(A)}{n(S)} = \frac{{}^4C_1 \times {}^{13}C_9 \times {}^{39}C_4}{{}^{52}C_{13}}$$

8. If 8 chocolates are distributed among 11 students, find the chance that particular students receives 5 chocolates.

Sol: Since there is no condition on the distribution of chocolates, each of the 11 students can get all 8 chocolates i.e. repetition is allowed.

Let S = Each of 11 students can get each of 8 chocolates

$\therefore$ n (S) = $11 \times 11 \times$ ...... 8 times.

$= 11^8$

Let event A

= A particular student gets 5 chocolates.

= After a student gets 5 chocolates can get each of remaining 3 chocolates.

$\therefore$ n (A) = $1 \times 10 \times 10 \times 10$

$= 10^3$

$$\therefore\ P(A) = \frac{n(A)}{n(S)} = \frac{10^3}{11^8}$$

9. 25 persons were invited for a party by host. What is the probability that two particular persons be seated on either side of the host at a circular table?

Sol: Including the host there are 26 person for a party.

∴ Let S = To arrange 26 at a circular table in 26 seats.

∴ n (S) = (26 – 1)! = 25!

Let event A

= Two particular persons are seated on either side of the host.

= Remaining 23 persons + the host can be seated in 24 seats and two particular persons can be seated on 2 seats either side of the host.

∴ n (A) = (24 – 1)! 2! = 23! 1!

$$\therefore\ P(A) = \frac{n(A)}{n(S)} = \frac{23!2!}{25!} = \frac{1}{300}$$

10. A computer software company is bidding for computer programs A and B. The probability that the company will get software A is 3/5, the probability that the company will get software B is 1/3 and the company will get both the software is 1/8. What is the probability that the company will get at least one software?

Sol: Let event A = The company gets software A

B = The company gets software B

Then A ∩ B = The company gets both software.

Given ; Their probabilities are:

$$P(A) = \frac{3}{5},\ P(B) = \frac{1}{3} \text{ and } P(A \cap B) = \frac{1}{8}$$

Probabilities that the company gets at least one software is.

P (A ∪ B) = P (A) + P (B) – P (A ∩ B)

$$= \frac{3}{5} + \frac{1}{3} - \frac{1}{8} \quad = \frac{72+40-15}{120}$$

$$= \frac{97}{120}$$

11. A bag contains 10 white balls and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that

a. First is white and second is black?

b. One is white and other is black?

Sol: The bag contains 25 balls, Since two balls are drawn in succession without replacement, for the conditional second event a ball is reduced in the bag.

Let event A = A white ball is selected.

Event B = A black ball is selected.

$\therefore$ B / A = Second ball is black, first ball being white.

A / B = Second ball is white, first ball being black.

(a) Probability that first ball is white and second is black

= P (event A and event B / A)

= P (A). P (B / A)

$$= \frac{10}{25} \times \frac{15}{24}$$

$$= \frac{1}{4}$$

(b) Probability that one ball is white and the other is black.

= P (First ball is white, second is black or First ball is black, second is white)

= P (event A and event B / A or event B and event A / B)

= P (A). P (B / A) + P (B), P (A / B)

$$= \frac{10}{25} \times \frac{15}{24} + \frac{15}{25} \times \frac{10}{24}$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

12. Three unbiased dice are tossed simultaneously. What is the probability that the number on the first two dice is greater than the number on the third die given that sum of the three numbers shown is 7?

Sol: Let S = Three unbiased dice are tossed simultaneously

= Set if triplets obtained from one face number from first, second, third die from the set {1, 2, 3, 4, 5, 6}

$\therefore$ n (S) = 216 triplets

Given that

Event A = The sum of all three numbers obtained is 7, the event has already occurred.

= {(1. 1, 5), (1. 2, 4), (1, 3, 3), (1, 4, 2), (1, 5, 1), (2, 1, 4)
(2, 2, 3), (2, 3, 2), (2, 4, 1), (3, 1, 3), (3, 2, 2), (3, 3, 1)
(4, 1, 2), (4, 2, 1), (5, 1, 1)}

$\therefore$ n (A) = 15 triplets

As the event A has already occurred, in the next event S is the sample space, A becomes sample space.

Required probability of event B such that A has already occurred where Event B = The numbers on first and second die are greater than number on the third die.

$\therefore$ Event (B / A) = A $\cap$ B

= The number on first and second die are greater than the number on the third die given that the sum of all there is 7.

= {(2, 4, 1), (3, 3, 1), (4, 2, 1)}

$\therefore$ n (B / A) = n (A $\cap$ B) = 3

$$\therefore P(B/A) = \frac{n(A \cap B)}{n(A)} = \frac{3}{15}$$

$$= \frac{1}{5}$$

13. Urn I contains 3 white, 2 black and 2 green balls, urn II contains 2 white, 3 black and 4 green balls; Urn III contains 5 white, 2 black and 2 green balls An urn is chosen at random and two balls are drawn, they happen to be black and green. What is the probability that they come from urns I, II or III?

Sol: Let events E_1 E_2, E_3 = Urn I, II or III is selected

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let A = A black ball and a green ball are selected from the selected urn.

Urn I contains 3 white, 2 black, 2 green (7) balls.

$$\therefore P(A/E_1) = \frac{{}^2C_1 \times {}^2C_1}{{}^7C_2} = \frac{2\times2}{\frac{7\times6}{2\times1}} = \frac{2\times2\times2}{7\times6}$$

$$= \frac{4}{21}$$

Urn II contains 2 white, 3 black, 4 green (9) balls.

$$\therefore P(A/E_2) = \frac{{}^3C_1 \times {}^4C_1}{{}^9C_2} = \frac{3\times4}{\frac{9\times8}{2\times1}} = \frac{3\times4\times2}{9\times8}$$

$$= \frac{1}{3}$$

Urn III contains 5 white, 2 black, 2 green (9) balls.

$$\therefore P\left(A / E_3\right)=\frac{{}^2C_1\times{}^2C_1}{{}^9C_2}=\frac{2\times2}{\frac{9\times8}{2\times1}}=\frac{2\times2\times2}{9\times8}$$

$$=\frac{1}{9}$$

$\therefore$ P (A) = P (E_1), p (A / E_1) + P (E_2), P (A / E_2) + P (E_3), P (A / E_3)

$$=\frac{1}{3}\times\frac{4}{21}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{9}$$

$$=\frac{1}{3}\left[\frac{4}{21}+\frac{1}{3}+\frac{1}{9}\right]=\frac{1}{3}\times\frac{12+21+7}{63}$$

$$=\frac{1}{3}\times\frac{40}{63}=\frac{40}{189}$$

Required probability that the balls are from Urn I

$$\therefore P\left(E_1/A\right)=\frac{P\left(E_1\right).P\left(A/E_1\right)}{P\left(A\right)}=\frac{\frac{1}{3}\times\frac{4}{21}}{\frac{40}{189}}$$

$$=\frac{\frac{1}{3}\times\frac{4}{21}}{\frac{1}{3}\times\frac{63}{40}}=\frac{4}{21}\times\frac{63}{40}=\frac{3}{10}$$

Similarly,

$$\therefore P\left(E_2/A\right)=\frac{P\left(E_2\right).P\left(A/E_2\right)}{P\left(A\right)}=\frac{\frac{1}{3}\times\frac{1}{3}}{\frac{1}{3}\times\frac{40}{63}}$$

$$=\frac{1}{3}\times\frac{63}{40}\qquad=\frac{21}{40}$$

$$\therefore P\left(E_3/A\right)=\frac{P\left(E_3\right).P\left(A/E_3\right)}{P\left(A\right)}=\frac{\frac{1}{3}\times\frac{1}{9}}{\frac{1}{3}\times\frac{40}{63}}$$

$$=\frac{1}{9}\times\frac{63}{40}\qquad=\frac{7}{40}$$

14. A doctor is called to see a sick child. The doctor has prior information that 80% of sick children in that area have the flu, while the other 20% are sick with measles. Assume that there is no other disease in that area. A well – Known symptom of measles is a rash. From the past record it is known that, chances of having rashes given that sick child is suffering from measles is 0.95. However, occasionally children with flu also develop rash, whose chances are 0.08. Upon examining the child. The doctor finds a rash. What is the probability that the child has measles?

Sol: Let event E_1 = A child in the area has flu.

$\therefore\ P(E_1)=80\% = \frac{80}{100} = \frac{4}{5}$

Event E_2 = A child in that area has measles.

$\therefore\ P(E_2) = 20\% = \frac{20}{100} = \frac{1}{5}$

Let event A = A child is examined to have rash.

Given that

Probability that the child has developed rash, having flu is $P(A / E_1) = 0.08$

And probability that the child has developed rash,

Having measles is $P(A / E_2) = 0.95$

$\therefore P(A) = P(E_1), P(A / E_1) + P(E_2), P(A / E_2)$

$$= \frac{4}{5}\times 0.08 + \frac{1}{5}\times 0.95$$

$$= \frac{0.32+0.95}{5} = \frac{1.27}{5}$$

Required probability that a child having measles has developed rash is

$$P(E_2/A) = \frac{P(E_2).P(A/E_2)}{P(A)} = \frac{\frac{1}{5}\times 0.95}{\frac{1.27}{5}}$$

$$= \frac{0.95}{1.27} \qquad = \frac{95}{127}$$

$$= 0.748$$

15. Suppose that it is odds against a person who is now 40 years of age living till he is 75are 5:3 and against a person B now 35 living till he is 70 is 11:5, find the chance that at least one of these persons will be alive 35 years hence.

Sol: Given odds against a person A now 40 years living till 75 are 5: 3

i.e. Odds against A being live after 35 years are 5:3.

i.e. P (A’): P (A) = 5:3

$\therefore\ P(A) = \frac{3}{8}$ and $P(A') = \frac{5}{8}$

Also, given odds against a person now 35 years living till 70 are 11:5

i.e. Odds against B being alive after 35 years are 11:5.

i.e. P (B’) = P (B) = 11: 5

$\therefore\ P(B) = \frac{5}{16}$ and $P(B') = \frac{11}{16}$

Clearly event A and B are mutually independent. Hence, A’ and B’ are also independent.

Required Probability,

35 years hence, at least one of A or B is alive

P (at least one is alive)

= 1 – P (none of A and B is alive)

= 1 – P (A’ and B’)

= 1 – P (A’). P (B’)

$= 1 - \frac{5}{8} . \frac{11}{16} = 1 - \frac{55}{128}$

$= \frac{73}{128}$

16. The odds that A speaks truth are 3:2 and odds that B speaks the truth are 5:3. In what percentage of cases are they likely to contradict each other on an identical point?

Sol: Odds in favour of A speaking truth are 3: 2

i.e. P (A), P (A’) = 3: 2

i.e. $P(A) = \frac{3}{5}$ and $P(A') = \frac{2}{5}$

Odds in favour of B speaking truth are 5: 3

i.e. P (B): P (B’) = 5: 3

$\therefore P(B) = \frac{5}{8}$ and $P(B') = \frac{3}{8}$

Naturally, A and B are speaking independently on identical point.

Required Probability that they contradict each other

= P (one speaks truth, other does not)

= P (A and B’ or A’ or B)

= P (A ∩ B’) + P (A’ ∩ B)

= P (A), P (B’) + P (A’), P (B)

$= \frac{3}{5} . \frac{3}{8} + \frac{2}{5} . \frac{5}{8} = \frac{9+10}{40} = \frac{19}{40}$

i.e. A and B are likely to contradict each on an identical point in

$= \frac{19}{40} \times 100$

$= \frac{190}{4}$

$= 47.5\%$ cases

17. A fair die is tossed twice. What are the odds in favour of getting 4,5 or 6 on the first toss and a 1, 2, 3 or 4 on the second toss?

Sol: Let S = A fair is tossed twice.

= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore$ n (S) = 36.

Let event A = First die shows 4, 5 or 6 and second die shows 1, 2, 3 or 4

= {(4, 1), (4, 2), (4, 3), (4, 4), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4)}

$\therefore$ n (A) = 12

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{12}{36} = \frac{1}{3}$$

$$\therefore P(A') = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$$

$\therefore$ Odds in favour of event A are

P (A): p (A') = 1: 2

18. Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that equation will have real roots.

Sol: Required to determine a, b and c by throwing a die each time so that the equation $ax^2 + bx + c = 0$ has real roots.

$\therefore$ S = A die is thrown three times, so that each throw gives values for a, b and c respectively.

$\therefore$ $n(S) = 6^3 = 216$ triplets (a, b and c).

To obtain the triplet (a, b, c) so that equation $ax^2 + bx + c = 0$ has real roots.

$\therefore$ The condition for real roots is $b^2 - 4ac \geq 0$.

Now $a, b, c \in \{1, 2, 3, 4, 5, 6\}$

$b \neq 1$ .... (because $b^2 - 4ac < 0$ when $b = 1$.)

Hence $b \notin \{2, 3, 4, 5, 6\}$.

Let event A = To obtain the triplet (a, b, c) such that $b > 1$ and

$b^2 - 4ac \geq 0$.

Here, we can enlist all the triplets required with

a = 1, 2, 3, 4, 5, 6; b = 2, 3, 4, 5, 6;

and c is required third value with talking a and b in order.

$\therefore$ A= {(1, 2, 1), (1, 3, 1), (1, 3, 2), (2, 3, 1), (1, 4, 1),
(1, 4, 2), (1, 4, 3), (1, 4, 4), (2, 4, 1) (2, 4, 2),
(3, 4, 1), (4, 4, 1), (1, 5, 1), (1, 5, 2), (1, 5, 3),
(1, 5, 4), (1, 5, 5), (1, 5, 6), (2, 5, 1), 2, 5, 2),

(2, 5, 3), (3, 5, 1), (3, 5, 2), (4, 5, 1), (5, 5, 1),
(6, 5, 1), (1, 6, 1), (1, 6, 2), (1, 6, 3), (1, 6, 4),
(1, 6, 5), (1, 6, 6), (2, 6, 1), (2, 6, 2), (2, 6, 3),
(2, 6, 4), (3, 6, 1), (3, 6, 2), (3, 6, 3), (4, 6, 1),
(4, 6, 2), (5, 6, 1), (6, 6, 1)}

n(A) = 43 triplets

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{43}{216}$

19. Three groups of children contain respectively 3 girls & 1 boy, 2 girls & 2 boys and 2 girl & 3 boys. One child is selected at random from each group. What is the chance that three selected consists of 1 girl and 2 boys?

Sol: Let the groups be

I $\rightarrow$ 3 girls, 1 boy

II $\rightarrow$ 2 girls, 2 boys

III $\rightarrow$ 1 girl, 3 boys

Let A = To select a girl from group I

A' = To select a boy

$\therefore P(A) = \frac{3}{4}$; $P(A') = \frac{1}{4}$

Similarly if B and C are for groups II and III

Then $P(B) = \frac{2}{4}$; $P(B') = \frac{2}{4}$

And $P(C) = \frac{1}{4}$; $P(C') = \frac{3}{4}$

A, B, and C are mutually independent, so also A', B' and C'.

Let X = To select 1 girl and 2 boys.

= A, B', C' or A', B, C' or A', B', C.

$\therefore$ P(X) = $P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$= P(A) \cdot P(B) \cdot P(C') + P(A') \cdot P(B) \cdot P(C') + P(A') \cdot P(B') \cdot P(C)$

$= \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{3}{4} + \frac{1}{4} \cdot \frac{2}{4} \cdot \frac{3}{4} + \frac{1}{4} \cdot \frac{2}{4} \cdot \frac{1}{4}$

$= \frac{18 + 6 + 2}{64}$

$= \frac{26}{64}$

$= \frac{13}{32}$

20. A room has 3 sockets for lamps. From a collection of 10 light bulbs 6 are defective. A person selects 3 at random and puts them in every socket. What is the probability that the room will be lit?

Sol: There are 10 bulbs in a collection.

S = To select 3 bulbs at random.

n(S) = ${}^{10}C_3$ $= \dfrac{10 \cdot 9 \cdot 8}{3 \cdot 2.1} = 120$

Let A = The room gets light.

At least one of the 3 bulbs selected is good.

Hence, it is convenient to consider the compliments of A.

A' = The room does not get light.

= All 3 bulbs selected are not good. (from 6.)

$\therefore$ n(A') = ${}^{6}C_3$ $= \dfrac{6.5.4}{3.2.1} = 20$

$\therefore$ P(A') = $\dfrac{n(A')}{n(S)} = \dfrac{20}{120} = \dfrac{1}{6}$

$\therefore$ Required Probability.

P(A) = 1 – P(A')

$= 1 - \dfrac{1}{6} = \dfrac{5}{6}$

21. A box contains 25 tickets numbered 1 to 25. Two tickets are drawn at random. What is the probability that the product of the numbers is even?

Sol: S = To select 2 tickets from the box containing 25 tickets.

n(A) = ${}^{25}C_2 = \dfrac{25 \cdot 24}{2 \cdot 1} = 25 \times 12 = 300$

Let A = The product of the numbers on 2 tickets is even.

= Both tickets bear even number or one ticket bears odd and the other bears even number.

= To select 2 tickets with even numbers (one of 12 tickets) or one ticket with even number (from 12) and the other ticket with odd number (from 13).

$\therefore$ n(A) = ${}^{12}C_2 + {}^{12}C_1 \times {}^{13}C_1$

$= \dfrac{12.11}{2.1} + 12 \times 13$

= 66 + 156

= 222

$\therefore$ P(A) $= \dfrac{n(A)}{n(S)} = \dfrac{222}{300}$

$= \frac{37}{50}$

22. If A, B and C are mutually exclusive and exhaustive events associated with the random experiment. Find P(A), given that $P(A) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$

Sol: Given events A, B, C are mutually exclusive and exhaustive in a sample space

$\therefore P(A) + P(B) + P(C) = 1$ ....(I)

Also given : $P(B) = \frac{3}{2}P(A)$

$P(C) = \frac{1}{2}P(B) = \frac{1}{2}\cdot\frac{3}{2}P(A) = \frac{3}{4}P(A)$

Substituting P(B) and P(C) in terms of P(A) in (I)

$P(A) + \frac{3}{2}P(A) + \frac{3}{4}P(A) = 1$

$\left(1 + \frac{3}{2} + \frac{3}{4}\right)P(A) = 1$

$\frac{4+6+3}{4}\cdot P(A) = 1$

$\frac{13}{4}P(A) = 1$

$\therefore P(A) = \frac{4}{13}$

23. The odds against a certain event are 5 : 2 and odds in favour of another chance independent event are 6 : 5. Find the chance that at least one of the events will happen.

Sol: Let A and B be the mutually independent events.

$\therefore P(A)\cdot P(B) = P(A \cap B)$

Given : Odds against event A are 5 : 2

$\therefore$ Odds in favour of event A are 2 : 5

$\therefore P(A) = \frac{2}{7}$

and odds in favour of event B are 6:5

$\therefore P(B) = \frac{6}{11}$

$\therefore P(A \cap B) = P(A)\cdot P(B)$

$= \frac{2}{7}\cdot\frac{6}{11}$

$= \frac{12}{77}$

Required probability

$= P(\text{at least one of the events occur})$

$= P(\text{either A occurs or B occurs or both occur})$

$= P(A \cup B)$

By Addition theorem

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{7} + \frac{6}{11} - \frac{12}{77}$$

$$= \frac{22 + 42 - 12}{77}$$

$$= \frac{52}{77}$$

24. Two-third of the students in a class are boys and rest are girls. It is known that the probability of a girl getting first class is 0.25 and that of boy getting is 0.28. Find the probability that a student chosen at random will get first class.

Sol: Let A = Selecting a boy from the class

$\therefore$ A' = Selecting a girl from the class

From the given ratio of boys and girls in the class, we can assume that

$$P(A) = \frac{2}{3} \quad \text{and} \quad P(A') = \frac{1}{3}$$

Let B = A student has got first class.

Then events

B /A = A student getting first class is a boy.

Given $P(B / A) = 0.28$

Similarly,

B / A' = A student getting first class is a girl.

Given $P(B / A') = 0.25$

Required Probability

= P (A student selected at random, getting first class)

= P (A and B or A' and B)

$= P(A \cap B) + P(A' \cap B)$

$= P(A).\, P(B/A) + P(A').\, P(B/A')$

… by Conditional Probability.

$$= \frac{2}{3}\times 0.28+\frac{1}{3}\times 0.25$$
$$= \frac{0.56}{3}+\frac{0.25}{3}$$
$$= \frac{0.81}{3}$$
$$= 0.27$$

25. A number of two digits is formed using the digits 1, 2, 3 ...., 9. What is the probability that the number so contain is even and less than 60?

Sol: S = A two digit number is formed using

Digits 1, 2, 3, 4, 5, 6, 7, 8, 9 without any precondition

$\therefore$ n (A) = ${}^9P_1 . {}^9P_1$ = $9 \times 9 = 81$

Let A = The two digit number is < 60.

i.e. First place can be filled by any one digit from 1, 2, 3, 4, 5 and since the number is even, second place can be filled by any digit from 2, 4, 6, 8,

$\therefore$ n (A) = ${}^5P_1 \times {}^4P_1 = 5 \times 4 = 20$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{20}{81}$

26. A card is drawn from a pack of well-shuffled 52 cards and a gambler bets that it is a spade or an ace. What are the odds against his winning this best?

Sol: S = A card is drawn at random from a well

Shuffled pack of 52

$\therefore$ n (S) = ${}^{52}C_1 = 52$

Let A = The gambler bets it is a spade,

= A spade is drawn from 13.

n (A) = ${}^{13}C_1 = 13$

$\therefore P(A) = \frac{13}{52}$

B = The gambler bets it can be an ace.

= The ace is drawn from 4.

n (B) = ${}^4C_1 = 4$

$\therefore P(B) = \frac{4}{52}$

A $\cap$ B = The card is a spade – ace

$\therefore$ n (A $\cap$ B) = 1

$\therefore$ P (A $\cap$ B) = $\frac{1}{52}$

By addition theorem

P (A $\cup$ B) = P (A) + P (B) – P (A $\cap$ B)

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{16}{52} = \frac{4}{13}$

i.e. P (the card is a spade or an ace) $= \frac{4}{13}$

P (the card not a spade and an ace) $= 1 - \frac{4}{13}$

$= \frac{9}{13}$

$\therefore$ Odds for this event are 4 : 9

$\therefore$ Odds against the gambler winning his bets are 9 : 4

27. A die is thrown twice and sun of the numbers appearing is observed to be 7. What is the conditional probability that the number 2 has appeared at least once?

Sol: S = A die is thrown twice

i.e. a set of 36 ordered pairs.

$\therefore$ n (S) = 36

Let A = The sum of numbers on the face is 7.

= {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$\therefore$ n(A) = 6

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

B = Number 2 appears at least once.

= {(1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (4, 2), (5, 2), (6, 2)}

A $\cap$ B = {(2, 5), (5, 2)}

n (A $\cap$ B) = 2

$\therefore$ $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

Required probability is

P (Event B occurs after A has occurred)

$$= P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/18}{1/6}$$

$$= \frac{1}{3}$$

28. In a bolt factory, Machines A, B and C manufacture respectively 25, 35 and 40 percent of the total. Out of their total output 5, 4 and 2 percent are defective. A bolt drawn from the procedure at random is found to be defective. What is the probability that it is manufactured by

i. machine A   ii. machine C?

Sol: In a factory producing bolts, three machines A, B, and C manufacture respectively 25%, 35% and 40% of the output of bolts.

A = a bolt is produced by machine A

B = a bolt is produced by machine B

C = a bolt is produced by machine C

Hence given their probabilities

$$P(A) = \frac{25}{100}; P(B) = \frac{35}{100}; P(C) = \frac{40}{100}$$

Let X= A defective bolt is chosen.

Given percentages of defective bolts from machines A, B, and C respectively are 5%, 4% and 2%.

$\therefore$ X / A = A defective bolt is from A

X / B = A defective bolt is from B

X / C = A defective bolt is from C.

Given their probabilities

$$P(X/A) = \frac{5}{100}; \qquad P(X/B) = \frac{4}{100}$$

$$P(X/C) = \frac{2}{100}$$

Required Probabilities

(i) P (Defective bolt is from A)

= P (the bolt is selected defective coming from machine A)

= P (A / X)

By Byes Theorem

$$P\left(A / X\right) = \frac{P(A).P(X/A)}{P(A).P(X/A)+P(B).P(X/B)+P(C).P(X/C)}$$

$$= \frac{\frac{25}{100} \times \frac{5}{100}}{\frac{25}{100}\times\frac{5}{100}\times\frac{35}{100}\times\frac{4}{100}\times\frac{40}{100}\times\frac{2}{100}}$$

$$= \frac{125}{125+140\ 80}$$

$$= \frac{125}{345}$$

$$= \frac{25}{69}$$

$$= 0.36$$

(ii) P (Defective bolt coming from C)

= P (C / X)

By Byes Theorem

$$P\left(C / X\right) = \frac{P(C).P(X/C)}{P(A).P(X/A)+P(B).P(X/B)+P(C).P(X/C)}$$

$$= \frac{\frac{40}{100}\times\frac{2}{100}}{\frac{25}{100}\times\frac{5}{100}+\frac{35}{100}\times\frac{4}{100}+\frac{40}{100}\times\frac{2}{100}}$$

$$= \frac{80}{125+140+80}$$

$$= \frac{80}{345} \quad = \frac{16}{69}$$

$$= 0.23$$

29. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a four. Find the probability that it is actually a four.

Sol: Let event A = The man speaks truth.

Given that he speaks truth 3 times out of 4

$\therefore\ P(A) = \frac{3}{4}$

Let S = A die is thrown.

= {1, 2, 3, 4, 5, 6}

$\therefore$ n (S) = 6

Let event B = The man declares that he has got a score of 4

= {4}

$\therefore$ n (S) = 1

$\therefore\ P(B) = \frac{n(B)}{n(S)} = \frac{1}{6}$

$\therefore$ Event B' = He has not got the score of 4.

$\therefore\ P\ (B') = 1 - P(B) = 1 - \frac{1}{6} = \frac{5}{6}$

Required Probability that

P (The man actually has got a score of 4)

= 1 – P (he says and has not got 4)

= 1 – P (A and B')

= 1– P (A $\cap$ B')

= 1 – P (A). P (B')

.... A and B' are mutually independent.

$= 1 - \frac{3}{4} . \frac{5}{6}$

$= 1 - \frac{5}{8}$

$= \frac{3}{8}$

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